Influence of Changes in Boundary Conditions on Bridge Response

P.J. Barr, A.M.ASCE[1]; M.W. Halling, ASCE[2]; and K.C. Womack, ASCE[3]

Abstract:

A curved, three-span continuous, steel I-girder bridge in Salt Lake City, Utah was tested in order to determine its dynamic and static load carrying properties for three boundary condition states. For each of the three boundary condition states, two dynamic forced vibration methods were applied to the bridge as well as a static live-load test. Velocity transducers, accelerometers, and strain gages were utilized to record the response of the bridge. The analysis and compilation of recorded dynamic response of the bridge enabled the preparation of mode shapes and natural frequencies for each boundary condition. This paper discusses the resulting changes in relevant dynamic properties and compares them with the changes in the static properties that were determined from the bridge response recorded from the live load tests.

---

[1] Associate Professor, Department of Civil and Environmental Engineering, Utah State University, 4110 Old Main Hill, Logan, UT, 84332-4110.
[2] Associate Professor, Department of Civil and Environmental Engineering, Utah State University, 4110 Old Main Hill, Logan, UT, 84332-4110.
[3] Professor, Department of Civil and Environmental Engineering, Utah State University, 4110 Old Main Hill, Logan, UT, 84332-4110.

Introduction

Steel, curved-girder bridges are a common solution to a wide variety of traffic routing and infrastructural needs. They provide for quick erection (in comparison to a multispan straight-girder bridge where multiple piers are required) and are extremely flexible in their application. Due to these benefits, the use of curved-girder bridges in our infrastructure has steadily increased over the last several decades. However, a better understanding of their in-service performance is still required. One of the largest research studies on the behavior of curved girder bridges was performed by the Federal Highway Administration (FHWA) under the Curved Steel Bridge Research Program (CSBRP) and funded by the (Zureick et al. 1999). This research endeavor was a large scale experimental and analytical program focusing on the improvement of design methods used for curved steel girder bridges. Other researchers, such as Huang et al. (1995), used load testing to identify the importance of centrifugal forces on curved bridges created by vehicle traffic. Most recently, DeSantiago et al. (2005) compared the analysis results of various curved and straight bridges and other researchers (Zhang et al. 2005; Samaan et al. 2005) investigated the lateral-load distribution of curved-girder bridges. A detailed review of the design of horiztonally curved I-girder bridges can be found in Linzell et al. (2004).

In addition to bridge live load studies, considerable effort has also been placed on quantifying a bridge's response when subjected to dynamic loads. During a modal testing program by Bolton et al. (2005), a concrete box girder bridge that had been periodically tested was significantly damaged by the Hector Mine California Earthquake. Because the bridge had been tested just weeks before the seismic forces, it provided an excellent pre and post event opportunity to evaluate a bridge structure. They found that the event caused an overall decrease in natural frequencies of 18% relative to pre-event values. Eberhard and Marsh (1997), performed progressive destructive testing on a reinforced concrete bridge by using six cycles of transverse displacements imposed on the bridge bents. For each progressive test the bridge bearing was weakened by excavating the soil behind the wingwalls and replacing the bridge bearing points with nylon blocks resting on greased, polished, stainless-steel plates. They found the overall system stiffness decreased by 91% comparing the "as is" condition with the fully damaged condition. Dynamic load testing of bridges has also been performed after stiffening repairs. Working on a four-span-arch bridge that had recently undergone stiffening repairs, Proulx et al. (1992) found that while some frequencies and modes were significantly altered others remained largely unchanged. Changes in modal properties have also been related to the amount of repair or damage a structure has experienced. Halling et al. (2001) tested a single span bridge in seven different damage states. A multi-span concrete bridge that underwent stiffening repairs did not see significant changes in natural frequencies largely due to the modest size of repairs performed (Salawu and Williams 1995). In a similar example, bearings were replaced on a 4-span concrete bridge with little effect noted to the modal properties and only a marginal increase in global stiffness (Salawu 1997).

When the I-15 reconstruction project in Salt Lake City, Utah called for the demolition of a curved-girder bridge a unique opportunity presented itself for testing an "as is" curved girder bridge. With the eventual replacement of this bridge, the Utah Department of Transportation (UDOT) allowed researchers the unique opportunity to test the effects that changes in boundary condition states had on the performance of a curved-girder bridge. A variety of testing methods were used for each boundary condition state including sinusoidal forcing, impact loading, and live-load testing. A summary of the boundary condition states and an outline of the methods used to collect bridge data for each condition state are presented. In addition, a comparison of the field test results from the sinusoidal testing, impact testing, and live load testing are presented. This comparison indicates promising testing procedures in the area of structural health monitoring.

Bridge Description

The curved-girder bridge, prior to the I-15 reconstruction, was designated as ramp A-6 over I-215 eastbound at the 6400 South interchange in Salt Lake City, Utah. Built in the early 1970's, this bridge served for many years as a connector between I-15 northbound and I-215 westbound traffic. An elevation view of the bridge superstructure is shown in Figure 1.

Figure 1. ELEVATION VIEW OF THE CURVED GIRDER BRIDGE

The bridge was designed as a noncomposite, three span, live-load continuous structure. The two exterior spans had lengths of 12.6 m (41' 6") and a center span length of 21.1 m (69' 3"). The designated survey and vertical profile line of the bridge had a radius of 145.6 m (477.5 ft). In addition to the horizontal curve, the alignment of the bridge also followed a vertical curve, which transitioned from a slope of -1.5% slope to -6.1% in 106.7 m (350 ft). The bridge deck was 12.9 m (42.3 ft) wide and was composed of 20.3 cm (8 in.) thick reinforced concrete that was superelevated at six percent. The bridge deck was constructed without joints and was integral with the approach slab for about 9 meters (30 feet) at both the north and south ends of the bridge. Strength tests were performed on several core samples from the bridge deck. These tests showed the deck concrete to have an average compressive strength (f'_c) of 35.1 MPa (5090 psi) which was significantly higher than the design strength of 20.7 MPa (3000 psi).

The bridge deck was supported on five continuous steel girders spaced at 2.69 m (8 ft 10 in.) on center. Girders one, two and three (from left to right) were identical in cross-section. Girders four and five had thicker flanges over the interior supports, but the same web thickness as the other girders. For live-load distribution factor purposes AASHTO refers to the exterior girders as G1 and G5 and the interior girders as G2 through G4. The same labeling was followed by the authors and will be used throughout this paper. All five girders had an increase in flange thickness until a longitudinal location of 9.14 m (30 ft) from their end bearing points to resist the negative bending moment. Diaphragms were placed at all bearing locations, at third points in the first and third spans, and at fifth points in the second or largest span. The intermediate diaphragms were constructed with partial depth, C 15 x 40 standard sections that were positioned radially to the girders. All supporting piers and abutments were aligned radially to the bridge. Additional bridge information can be found in Womack et al. (2001).

Prior to the load test, the girders were inspected and found to be in good condition, with no sign of deterioration. The steel girders were supported at four locations (two abutments and two interior piers) along their length on self-lubricating bronze plates. Three of these bearing connections were intended to prevent vertical and radial translations but allow longitudinal and rotational movement. The fourth bearing allowed rotations but inhibited translations in all other degrees of freedom. The bearings were found to be severely deteriorated and nonfunctioning. It was discovered that three of the interior supports were not functioning as designed. The flange plates at some supports had been welded to the steel channels beneath, effectively bypassing the bronze bearing plates and restricted any movement of the bearings. In addition, some horizontal shifting of the bridge on the abutments had taken place and had damaged the bearings.

Boundary Condition States

The dynamic forced vibration and static load testing conducted on this bridge were performed for three boundary condition states. The first state was the "as is" condition which included the welded and frozen bearings along with the integral approach slabs described earlier. This first test was used as a baseline measurement to compare the effects that changes in boundary conditions had upon the bridge behavior. The testing of the second boundary condition state was performed with the same bearing support conditions; however, both integral approach slabs were severed. This left a gap at both ends of the bridge which effectively "freed" the bridge deck from the approach slabs. The third boundary condition state consisted of reducing the translational and rotational resistance at the bearing points of the girders. The original abutment supports which consisted of welded bronze bearings were replaced with Teflon/stainless steel bearings at both ends of the bridge. Additionally, the welds on the intermediate bearings at the piers were removed and the bearings were greased. These boundary condition states provided a verifiable way of adjusting the bridge's boundary conditions in order to facilitate correlation between the support conditions and changes in the bridge's dynamic and static properties.

Forced Vibration Testing

Lateral load testing of the curved-girder bridge was accomplished through the application of forced vibration. Two separate methods of forced vibration testing were used; sinusoidal and impact forcing. Both of these methods were employed for each of the three boundary condition states to allow for a comparison of the bridge's dynamic response when subjected to lateral forces. An eccentric mass shaker was used to generate the sinusoidal force applied to the bridge. The shaker was capable of imparting a sinusoidal forcing function of up to 89 kN (20 kips) in any horizontal direction. The shaker was mounted on the bridge at a location where it would not coincide with a modal node. A "Radial" test was performed using the shaker to induce sinusoidal forcing in a direction perpendicular to the longitudinal axis of the bridge. The shaker was also used to perform a "Tangential" test which consisted of inducing a sinusoidal force in a direction parallel to the longitudinal axis of the bridge. Both of these sinusoidal forcing tests were performed with the eccentric-mass shaker applying the load to the structure in a frequency range from 0.5 Hz to 20 Hz in 0.02 Hz increments. Fig. 2 shows the location of where the shaker was applied to the test structure.

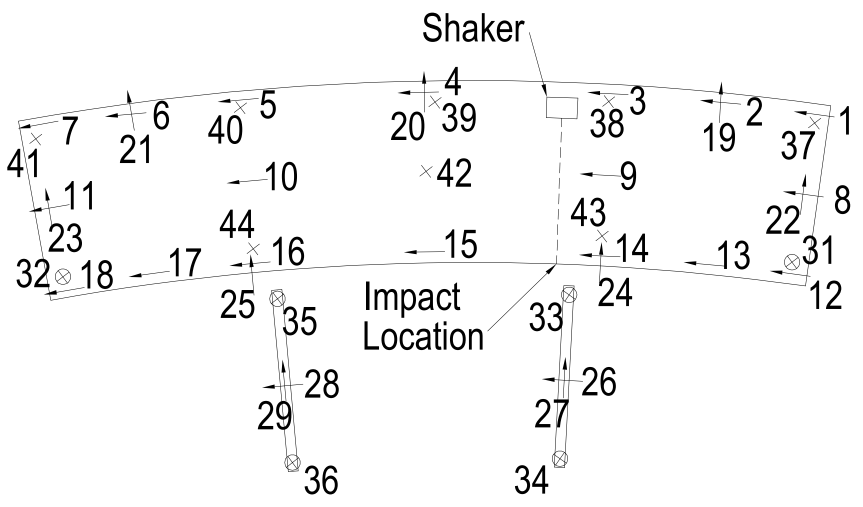

Figure 2. BRIDGE SENSOR LAYOUT

Striking the side of the bridge deck with a 2.5 kN (560 lb) hammer was used to generate an applied horizontal force during the “Impact” test. The location selected for striking the bridge was determined to be on the inside radius of the bridge directly along a radial line coinciding with the location of the eccentric mass shaker. The impact point of the hammer was perpendicular to the concrete deck of the bridge and located below the jersey barrier and above the steel girders. Each impact test consisted of striking the bridge 32 times to ensure an accurate recording had been taken. Sinusoidal (both radial and tangential) and impact testing were performed for each of the three boundary condition states.

To measure the bridge’s response due to the forced vibration, the structure was instrumented with thirty six 1-Hz velocity transducers and eight 1-g vertically oriented accelerometers for a total of 44 channels. Fig. 3 also shows the layout of the sensor array with instrumentation placed at the bearing points, mid-spans, and at selected locations along the centerline of the bridge. The number marks the sensor location and the arrow indicates the direction of the measured response. A 16-bit A/D card with anti-aliasing filters was used to collect the data from the sensor array.

Forced Vibration Testing Results and Evaluation

The first step in evaluating the raw data from the bridge was representing the bridge response due to varied forcing amplitudes and methods. The response for each forcing magnitude was normalized by the amplitude of the applied forcing function. Therefore all displacements are in units of length per unit force. In addition, a fast Fourier transform was performed to translate the data from the time domain into the frequency domain to facilitate identification of the natural frequencies. Once all the data were transformed and normalized the frequency response functions were plotted for each channel.

Figures 3 and 4 illustrate a sample of the frequency response function (FRF) plots of selected channels during State 1 for forcing frequencies ranging from 1 to 10 Hz. Plots of channels from both an impact (Fig. 3) and sinusoidal (Fig. 4) test are given. The natural frequencies in these plots are represented as peaks that correlate to the structures amplified resonance when resonating at a natural frequency. This approach has been used in several past studies (De Roeck et al. 2000). Some channels, because of their orientation or location, will not exhibit all frequencies. For example, a channel located parallel to the long axis of the bridge will likely miss a mode that contains movement largely perpendicular to the long axis of the bridge. Because of this, the overall natural frequencies reported are a synthesis of the frequencies found at each channel with only the strongest results reported. Consequently, four of the most representative channels were selected for the Fig. 3 and Fig. 4.

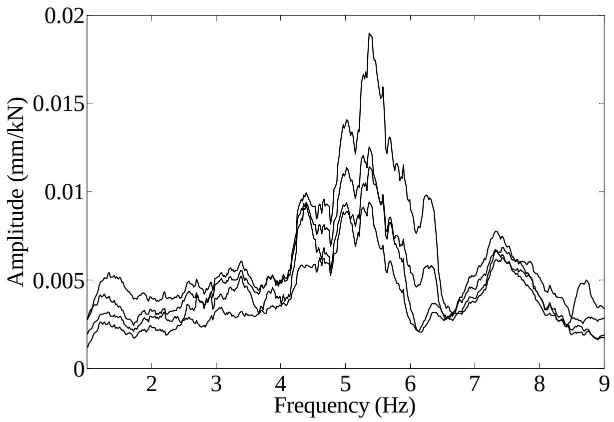

Figure 3. Combined FRF Plots for Impact Test State 1

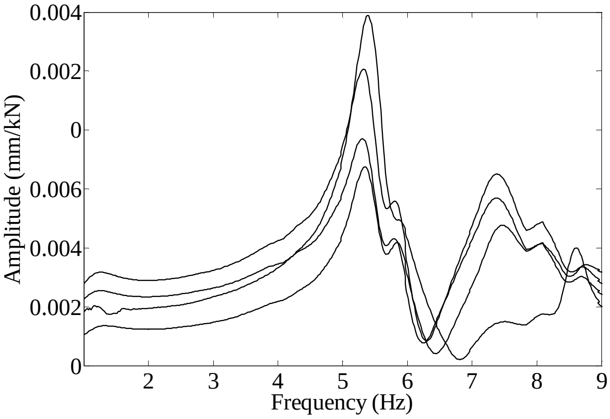

Figure 4. Combined FRF Plots for Sinusoidal Test State 1

Because of the large amount of data that needed to be analyzed, a database was constructed to compare the frequency response functions recorded at each channel for each boundary condition state. The natural frequencies resulting from each frequency response function were entered into this database and analyzed to determine the overall modal frequencies recorded for each boundary condition state of the bridge. As mentioned earlier, only natural frequencies that were exhibited in a large number of channels and were represented in both impact and sinusoidal testing methods are reported. Table 1 lists the first six modes as determined by this analysis.

In a comparison of the natural frequencies of the three states, Table 1 shows that as the restraint of the bridge was reduced the natural frequency was also reduced. In general, Modes 1 and 2 experienced the largest change in natural frequencies for the different boundary condition states. Reducing the restraint at the bearing supports (comparing State 2 to State 3) had the largest influence of the natural frequencies of the bridge reducing the natural frequencies for Modes 1 and 2 by more than 61% and 32% respectively. On average, severing the approach slab with a concrete saw from the bridge deck (comparing State 1 to State 2) reduced the natural frequencies by roughly 12% and replacing the bearings (comparing State 2 to State 3) reduced the natural frequencies by nearly 20%.

Table 1. Forced Vibration Natural Frequencies

Mode	State 1 (Hz)	State 2 (Hz)	State 3 (Hz)
1	4.44	3.84	1.49
2	5.42	4.37	2.95
3	5.96	5.61	5.20
4	7.34	6.09	5.60
5	8.00	6.73	6.27
6	8.72	8.49	8.48

Once the modal frequencies were determined, the corresponding mode shapes were then plotted using the response data at the natural frequencies listed in Table 1. The plots of the mode shapes provided valuable observations regarding the global behavior of the bridge. For State 1 the bridge responded with a greater amount of torsional movement than in States 2 or 3. This can be attributed to the frozen bearings, welded bearings, and integral approach slab previously discussed. These supports provided for very stiff end bearing conditions which were reflected in the mode shapes. The bridge's tendency to move torsionally in the "as is" state illustrates that the bearings and end conditions were not functioning as a pin and roller. The frozen bearings were stiff enough to force bending and rotation in the bridge structure before allowing sliding or rotation to occur at the bearing points.

For the second boundary condition state, the approach slabs were severed which provided the bridge with a greater ability to translate. While allowing greater translation, this change in boundary conditions also changed the measured mode shapes and their order of occurrence with respect to State 1. The first two modes remained as the first transverse bending and first torsional modes but switched order of occurrence. In addition, both modes decreased in frequency due to the changed boundary conditions. Additionally, the released boundary conditions allowed the bridge to begin exhibiting more rigid body movements. Modes 1 and 3 in the second boundary condition state moved laterally as a rigid body. On the whole, it can be stated that severing the approach slab profoundly altered the response of the bridge. From this, the importance of understanding a structure's boundary conditions and fixities can be illustrated. Changes in the connections within a structure or of the structure's bearing on surrounding surfaces can alter its response significantly.

Boundary condition changes from state 2 to state 3 also significantly altered the bridges response. Because of the reduction in restraint that was induced in this condition state the bridge translated enough to close the saw-cut gap between the deck and approach slab creating a contact point at the North abutment between the approach slab and the bridge deck. This contact occurred during both the radial and tangential sinusoidal forcing tests. In contrast, the closing of the gap did not occur during the impact test or the live-load test. Because of the difference in gap behavior, the mode shapes recorded during the two sinusoidal tests and the impact test were also different. Fig. 6 shows the difference in the recorded mode shapes for the radial and impact test, respectively. The contact on the north abutment caused the formation of a pivot point around which the entire bridge rotated during radial translational modes. The point of contact did not affect all modes in the response. Modes that were torsional or vertically dominant were not affected because the movement was not in the direction of the gap.

Two conditions are believed to be the cause of why the contact point was not manifested in the impact test. First, the amount of energy applied to the system during an impact test is much smaller than the energy applied during a sinusoidal test. In many cases, large amounts of information can reliably be extracted by applying reasonably small forces. However, as is illustrated with the results of this test, at times a small amount of energy can fail to extract certain behavior by not having the ability to isolate special structural responses that occur due to the vibration and resonance of the system. Secondly, during the sinusoidal testing the shaker starts by rotating at the lowest frequency for which recording will take place. From there the forcing frequency is increased in increments of 0.02 Hz. At each incremented frequency, the bridge is given 20-30 seconds for the steady state response to stabilize before data recording is initiated. Once begun, the data collection system will record response behavior for a period of approximately 20 seconds. The frequency of the shaker is then incremented higher and the monitoring process starts over again. This testing procedure allows for the effects of resonance to take place when the bridge is excited at its natural frequency. The bridge can be allowed to vibrate at or near its natural frequency for a number of minutes when taking into account the forcing in the frequency steps just before and after the natural frequency. This extended period of resonance allows for more bridge displacement and energy absorption. A sinusoidal test is therefore more likely to promote the formation of contact points than an impact test. This field observation is important when considering the changes in structural response as the level or type of forcing is altered.

Static Live Load Testing

In addition to determining modal properties of the bridge through forced vibration testing, a static live-load test was performed for each of the boundary condition states. Quantifying the effects of changing boundary conditions on the bridge live-load response was of interest to the Federal Highway Administration (FHWA) and Utah Department of Transportation (UDOT). The live load was applied by slowly driving one or two weighted trucks along three predetermined paths while instrumentation recorded the changes in strain at various locations along the bridge girders. The left wheel of the weighted truck was driven along each load path to apply the load to the desired location. The first load path was 0.43 m (1.4 ft) to the left of Girder 1. The second load path was 0.91 m (3 ft) on the left of Girder 3 and the third load path was 0.43 m (1.4 ft) to the left of Girder 5. These load paths were selected in order to apply a majority of the truck load(s) to the two exterior girders (Girders 1 and 5) and the center girder (Girder 3).

Strain gages developed by Bridge Diagnostic Inc. (BDI) were used to monitor the bridge performance under live load. In all, there were 136 instrumentation locations selected encompassing the girders in all three spans. However, because only 48 BDI reusable strain gauges were available at the time of testing, the tests were performed in three phases with the gauges attached on different bridge members each time until all the instrumentation locations had been monitored. Eight of the instrumentation locations that were used in Phase 1 were repeated in Phase 3 to confirm consistency between tests. A comparison of these gauges showed the strains discrepancy from the two phases were within 1%.

The longitudinal position of the weighted truck as it was driven along each load path was recorded by electronically counting the wheel revolutions of the front tire traveling along the load path from the south abutment, which was designated as the origin. The electronic device placed a marker in the data files, which was later used to convert to distances along the length of the bridge. When two trucks were used, the location of the truckloads was also related to the position of the front tire of the lead truck. When two trucks were tested in series, the second truck was chained to the first and was towed across the bridge. The driver of the second truck lightly applied the brake to keep the chain taught.

An analysis by Yanadori (2005) showed that for areas of the girders that experienced positive moments, the neutral axis based on strain distribution exhibited partial composite action between the girder and concrete deck. This is an important observation considering that the bridge was not designed for composite behavior. In contrast, it was also observed that in areas of predominately negative moment (near the interior piers), there was nearly no recorded composite action between the deck and girder. This change was attributed to the higher interface shear near the interior piers.

Fig. 5 shows the measured midspan strains at the bottom of Girder 1 for the first load path for each boundary condition state. The bottom strains were negative when the truck was in Spans 1 and 3 and positive when the truck was in Span 2. This trend was consistent for the other midspan gauge locations (Womack and Crookston, 2003). An analysis of the recorded strain data from the various live load tests, showed that going from boundary condition state 1 to 2 (removing the integral approach slab) slightly increased the positive and negative strains on the bridge with a maximum deviation of nearly 2%. This percent increase was significantly smaller than the average 12% decrease that was measured for the modal frequencies. Replacing the nonfunctioning bearings with new supports for boundary condition state 3 caused an additional 3% increase in the value of the positive and negative strains on the bridge when compared to the recorded strains of boundary condition state 2. This increase was also small in comparison to the average 20% change in modal frequencies measured during the dynamic test.

It was therefore apparent that while changing the boundary conditions of the bridge from what seemed like completely non-functioning bearings and integral approach slabs to a system with Teflon bearings and no deck attachment at the abutments caused only minimal changes in the resulting strains and therefore maximum bending moments in the girders. Therefore it was concluded that while changing boundary conditions had a significant influence on the dynamic bridge behavior, they had little effect on the live load response of the bridge. Yanadori (2005) suggests that these types of changes in boundary conditions could be reasonably neglected in design for live load conditions.

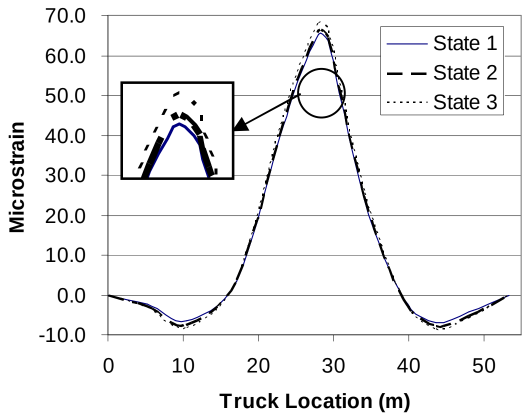

Figure 5. Strain Curves from Live Load Test

<u>Conclusions</u>

The reconstruction of the I-15 corridor through Salt Lake City, Utah provided an excellent opportunity for dynamic and static testing of a steel, curved-girder bridge under three different boundary conditions. Dynamic testing evolved using both sinusoidal as well as impact excitation with an array of velocity transducers and accelerometers to collect the response. The static testing was accomplished by means of a live load test with strain gages monitoring the bridge response. The measured performance of the bridge was analyzed resulting in the following conclusions:

- The results of the live-load testing showed that changing the boundary conditions increased the maximum moments on the bridge by only 5%. However, the dynamic testing under the same three boundary condition states produced changes in modal frequencies of up to 34%. Because small changes in modal properties are difficult to attribute to damage, the significantly larger changes in dynamic properties found in this study are an encouraging indication of the feasibility of structural health monitoring using dynamic techniques.
- A reduction in restraint stiffness resulted in a change in the order of modes for each of the tested boundary condition states. For example, changes in boundary conditions led to the introduction of modes that were not present in the original structure. This was verified by a MAC analysis between boundary condition states that indicated many modes changed shape or shifted their order of occurrence from state to state.
- The test results showed that a few modes increased in natural frequency when a specific mode shape was identified using MAC correlations to ensure similarity. This result verified initial expectations that as boundary conditions changed; altered mode shapes were expected.
- Evidenced in condition State 3, impact testing may not be suitable for testing certain types of structures because it cannot provide enough energy or time for resonance to promote the formation of contact points. A comprehensive testing program should include careful consideration of the strain levels of interest and a recognition of the limitations in small strain testing.

Acknowledgement

The authors would like to acknowledge the Federal Highway Administration (FHWA) and the Utah Department of Transportation (UDOT) for their generous support of the testing presented in this study. The authors would also like to thank Bridge Diagnostics Inc. of Boulder Colorado for their help during the live load portion of the study and Tyler Mertlich, Naoki Yanadori and Brad Crookston who worked on this project as part of their graduate studies.

References

Bolton, R., Sikorsky C., Park S., Choi S., and Stubbs N. (2005). “Modal Property Changes of a Seismically Damaged Concrete Bridge.” *ASCE Journal of Bridge Engineering,* 10(4), 415-428.

De Roeck, G., Peeters, G., and Ren, W.X. (2000). “Benchmark Study on System Identification Through Ambient Vibration measurements.” *Proceedings of the 18th International Modal Analysis Conference*, San Antonio, TX, 1225-1231.

DeSantiago, E., Mohammadi, J., and Albaijat, H.M.O. (2005). “Analysis of Horizontally Curved Bridges using Simple Finite-Element Models.” *ASCE Journal of Bridge Engineering*, 10(1), pp.18-21.

Eberhard, M. O., and Marsh, M. L. (1997). “Lateral-Load Response of a Reinforced Concrete Bridge.” *Journal of Structural Engineering*, 123(4), 451-460.

Halling, M. W., Muhammad, I., and Womack, K. C. (2001). “Dynamic field testing for condition assessment of bridge bents.” J. Struct. Engrg., 127(2), 161- 167.

Huang, D., Wang, T. L., and Shahawy, M. (1995). “Dynamic Behavior of Horizontally Curved I-Girder Bridges.” *Computer and Structures*, 57(4), 703-714.

Linzell, D., Hall, D., and White, D. (2004). "Historical Perspective on Horizontally Curved I Girder Bridge Design in the United States." ASCE Journal of Bridge Engineering, 9(3), pp. 218-229.

Proulx, J., Herbert, D., and Paultre, P. (1992). "Evaluation of the Dynamic Properties of a Steel Arch Bridge." *Proc., 10th Int. Modal Anal. Conf.,* Soc. For Experimental Mech., Bethel, Conn., 160-165.

Salawu, O.S. (1997). "Assessment of Bridges: Use of Dynamic Testing." *Canadian Journal of Civil Engineering*, 24, 218-228.

Salawu, O.S., and Williams, C. (1995). "Bridge Assessment Using Forced-Vibration Testing." *Journal of Structural Engineering*, 121(2), 161-173.

Samaan, M., Sennah, K. and Kennedy, J.B. (2005). "Distribution Factors for Curved Continuous Composite Box-Girder Bridges." ASCE Journal of Bridge Engineering 10(6), pp. 678-692.

Womack, K. and Crookston, B. (2003). "Method of Analysis Comparison Study for a Curved, Steel Girder Bridge." Utah Department of Transportation Research Report No. UT-03.02, Utah State University, Logan, Utah.

Yanadori, N. (2005). Live-Load Analysis of a Curved I-Girder Bridge. *Masters Thesis,* Utah State University, Logan, UT.

Zhang, H., Huang, D., and Wang, T.L. (2005). "Lateral Load Distribution in Curved Steel I-Girder Bridges." ASCE Journal of Bridge Engineering. 10(3), pp.281-290.

Zureick, A., Naqib, R., and Yadlosky, J. M. (1999). "Curved Steel Bridge Research Project, Interim Report I: Synthesis." Publication Number FHWA-RD-993-129, Federal Highway Administration, December.